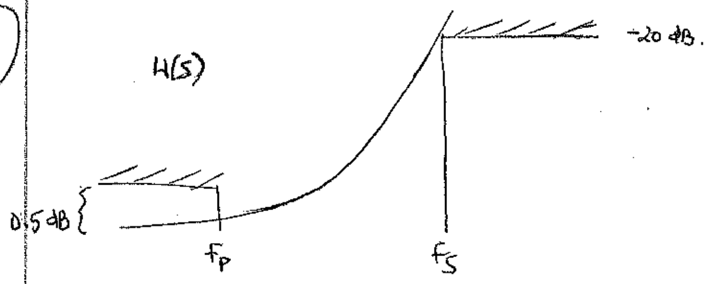
(1)



Low-pass filter.

Given: $f_p = 1\text{ KHz}$; $\alpha_p \le 0.5\text{dB}$

$f_s = 2\text{ KHz}$; $\alpha_s \ge 20\text{dB}$

Slectivity Parameter: $K = \frac{f_p}{f_s} = \frac{1\text{ KHz}}{2\text{ KHz}} = 0.5$

$$K^{2n} \le \frac{10^{\alpha_p/10} - 1}{10^{\alpha_s/10} - 1}$$

$$\Rightarrow K^n \le \sqrt{\frac{10^{0.05} - 1}{10^2 - 1}} = 0.035107$$

$$\Rightarrow n \ge \frac{\log(0.035107)}{\log(K)} = \frac{-1.4546048}{-0.30103} = 4.8321$$

Since n should be an integer number, set $n = 5$.

For a Butterworth filter, $\alpha(\omega) = 10\log(1 + |K|^2) = 10\log(1 + C_n\omega^{2n})$

$$\Rightarrow 10^{\alpha(\omega)/10} = 1 + C_n\omega^{2n}$$

$$\Rightarrow C_n = \frac{10^{\alpha_s/10} - 1}{\omega^{2n}} = \frac{10^{\alpha_s/10} - 1}{(2\pi f_s)^{2n}} = \frac{10^2 - 1}{(2\pi \times 2\times10^3)^{10}} = 1.008\times10^{-39}$$

This is a very small value of C_n. To avoid this, set $C_n = 1$. Now, find the 3-dB frequency accordingly. setting $\omega_{3\text{-dB}} = \omega_0$, the normalizing frequency

$$\Rightarrow \omega_0 = C_n^{-1/2n} = (1.008\times10^{-39})^{-1/10} = 7936.82 \text{ radians.}$$

In terms of normalized frequency $\Omega = \omega/\omega_0 \Rightarrow \omega = \Omega\omega_0$,

$$|K(j\Omega)|^2 = C_n(\omega_0\Omega)^{2n} = \Omega^{2n}$$

$\Rightarrow |K|^2 = \Omega^{10}$ and $K(s) = \pm S^5$ where, $S = j\Omega = \frac{s}{\omega_0}$ $\Rightarrow$ s-plane circle becomes unit circle

$$S_k = e^{j\pi(n-1+2k)/2n} = e^{j\pi(4+2k)/10} = e^{j\pi(0.4+0.2k)}$$

$$\left.\begin{aligned}
|S_1| &= |e^{j0.6\pi}| = -0.30902 + j0.951056\\
|S_2| &= |e^{j0.8\pi}| = -0.809 + j0.588\\
|S_3| &= |e^{j\pi}| = -1\\
|S_4| &= |e^{j1.2\pi}| = -0.809 - j0.588\\
|S_5| &= |e^{j1.4\pi}| = -0.30902 - j0.951056
\end{aligned}\right\}\text{ normalized poles.}$$

$$H(s) = \pm \prod_{k=1}^{5} (s - s_k) = (s-s_3)\left[(s-s_1)(s-s_5)\right]\left[(s-s_2)(s-s_4)\right]$$
$$= (s+1)\ [s^2 + 0.618 s + 1][s^2 + 1.618 s + 1]$$
$$= s^5 + 3.236068 s^4 + 5.236068 s^3 + 5.236068 s^2 + 3.236068 s + 1$$

So the normalized transfer function,

1

$$A_v(s) = \frac{1}{H(s)} = \frac{1}{s^5 + 3.236 s^4 + 5.236 s^3 + 5.236 s^2 + 3.236 s + 1}$$

Un-normalized transfer function,

$$A_v\left(s = \frac{s}{\omega_0}\right) = \frac{1}{\left(\frac{s}{\omega_0}\right)^5 + 3.236\left(\frac{s}{\omega_0}\right)^4 + 5.236\left(\frac{s}{\omega_0}\right)^3 + 5.236\left(\frac{s}{\omega_0}\right)^2 + 3.236\left(\frac{s}{\omega_0}\right) + 1}$$

$$= \frac{\omega_0^5}{s^5 + 3.236\,\omega_0 s^4 + 5.236\,\omega_0^2 s^3 + 5.236\,\omega_0^3 s^2 + 3.236\,\omega_0^4 s + \omega_0^5}$$

$$\Rightarrow \boxed{A_v(s) = \frac{C_0}{C_5 s^5 + C_4 s^4 + C_3 s^3 + C_2 s^2 + C_1 s + C_0}}$$

where,

$C_0 = \omega_0^5 = 3.1994 \times 10^{19}$

$C_1 = 3.236\,\omega_0^4 = 1.2841 \times 10^{16}$

$C_2 = 5.236\,\omega_0^3 = 2.6179 \times 10^{12}$

$C_3 = 5.236\,\omega_0^2 = 3.2984 \times 10^{8}$

$C_4 = 3.236\,\omega_0 = 2.5684 \times 10^{4}$

$C_5 = 1 = 1$

Poles and Zeros :

Zeros : none 5 at ∞

Poles :

Normalized → -1 ; $-0.309 \pm j0.951$; $-0.809 \pm j0.588$

Un-normalized → -7936.82 ; $-2452.5 \pm j7547.9$; $-6420.9 \pm j4666.9$
(multiply by ω_0)

(2)

(9)

Given: $f_p = 10KHz$ $\alpha_p = 0.5\,dB$

$f_s = 25KHz$ $\alpha_s = 50\,dB$

Selectivity parameter: $K = \frac{f_p}{f_s} = \frac{10K}{25K} = 0.4$

$$K_1 = \sqrt{\frac{10^{\alpha_p/10} - 1}{10^{\alpha_s/10} - 1}} = \sqrt{\frac{10^{0.05} - 1}{10^5 - 1}} = 0.0011 0462516$$

$$n \geq \frac{\cosh^{-1}(1/K_1)}{\cosh^{-1}(1/K)} = \frac{7.5014}{1.5668} = 4.788$$

$\Rightarrow n = 5$

From the Chebyshev normalized table, @ n=5;
zeros: none
Poles: -0.3623196; $-0.111963 \pm j\,1.01156$; $-0.293123 \pm j\,0.625177$ ← Natural Modes.

Now, $H(s) = \underbrace{2^{n-1} K_p}_{C} \underbrace{\left(s^n + \sum_{k=0}^{n-1} a_k s_k\right)}_{\text{coefficients and polynomial}}$

$$C = 2^{n-1} K_p = 2^4 \sqrt{10^{\alpha_p/10} - 1} = 2^4 \sqrt{10^{0.05} - 1} = 5.58898$$

$a_4 = 1.1724909$
$a_3 = 1.9373675$
$a_2 = 1.3095747$ $\Rightarrow$
$a_1 = 0.7525181$
$a_0 = 0.1789234$

$$H(s) = 5.589\left[s^5 + 1.1725 s^4 + 1.9374 s^3 + 1.3096 s^2 + 0.7525 s + 0.1789\right]$$

Normalized transfer function,

$$A_v(s) = \frac{1}{H(s)} = \frac{1}{5.589\left[s^5 + 1.1725 s^4 + 1.9374 s^3 + 1.3096 s^2 + 0.7525 s + 0.1789\right]}$$

Since $\omega_o = 2\pi f_p = 2\pi(10 \times 10^3) = 2\pi \times 10^4$

$$A_v\left(s = \frac{s}{\omega_o}\right) = \frac{0.1789\,\omega_o^5}{s^5 + 1.1725\,\omega_o s^4 + 1.9374\,\omega_o^2 s^3 + 1.3096\,\omega_o^3 s^2 + 0.7525\,\omega_o^4 s + 0.1789\,\omega_o^5}$$

$$A_v(s) = \frac{C_o}{C_5 s^5 + C_4 s^4 + C_3 s^3 + C_2 s^2 + C_1 s + C_o}$$

$C_o = 1.752 \times 10^{23}$
$C_1 = 1.1728 \times 10^{19}$
$C_2 = 3.2483 \times 10^{14}$
$C_3 = 7.6484 \times 10^{9}$
$C_4 = 7.3669 \times 10^{4}$
$C_5 = 1$

(2)

(b)

For $f = 25\,KHz$,

$$S = j\frac{\omega}{\omega_0} = j\frac{f}{f_p} = j\frac{25}{10} = j2.5$$

$$H(s=j2.5) = 5.589\left[j(2.5)^5 + 1.1725(2.5)^4 - 1.9374j(2.5)^3 - 1.3096(2.5)^2 + 0.7525j(2.5) + 1\right]$$

$$= 255.9766 - 45.745 + 1 + j\left[545.8 - 169.1875 + 10.5145\right]$$

$$= 211.2316 + j387.1278$$

$$\Rightarrow H(S) = 441.0065 \angle 1.0713\,rad. = 20\log(441) = 26.44\,dB$$

Attenuation Magnitude @ 25 KHz, $\boxed{H(s) = 52.9\,dB}$

(c)

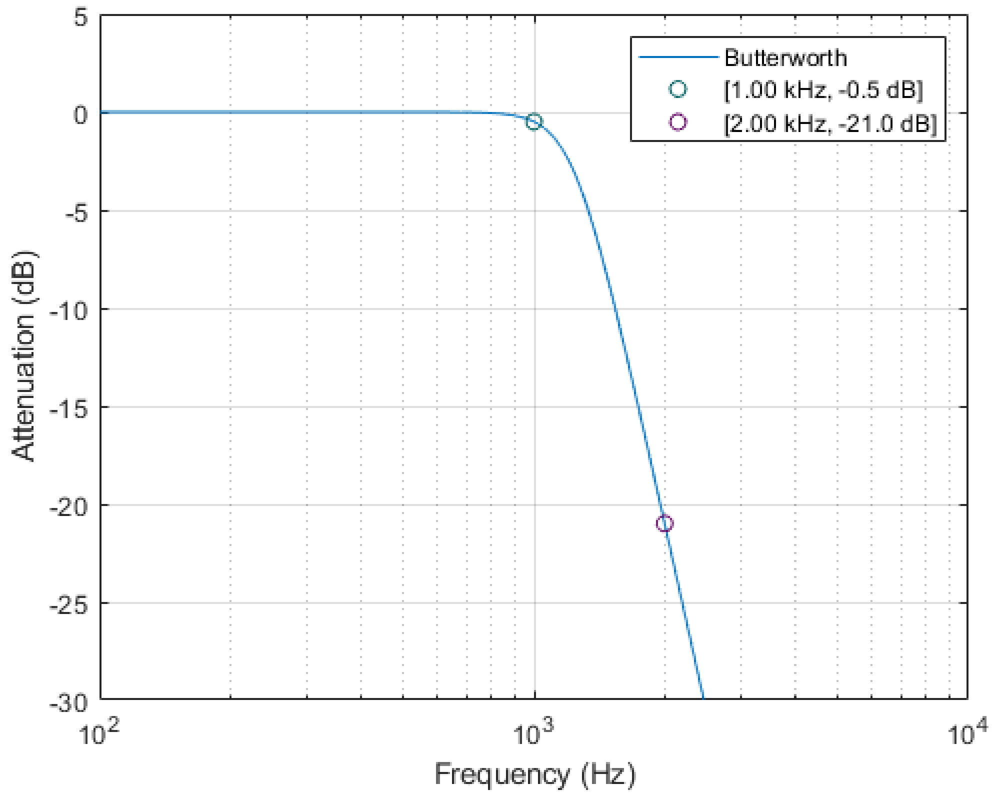
At $f = f_p = 10KHz$, $S = j\frac{\omega}{\omega_p} = j\frac{f}{f_p} = j1$

$$\therefore H(s=j) = 5.589\left[j + 1.1725 - 1.9374j - 1.3096 + 0.7525j + 1\right]$$

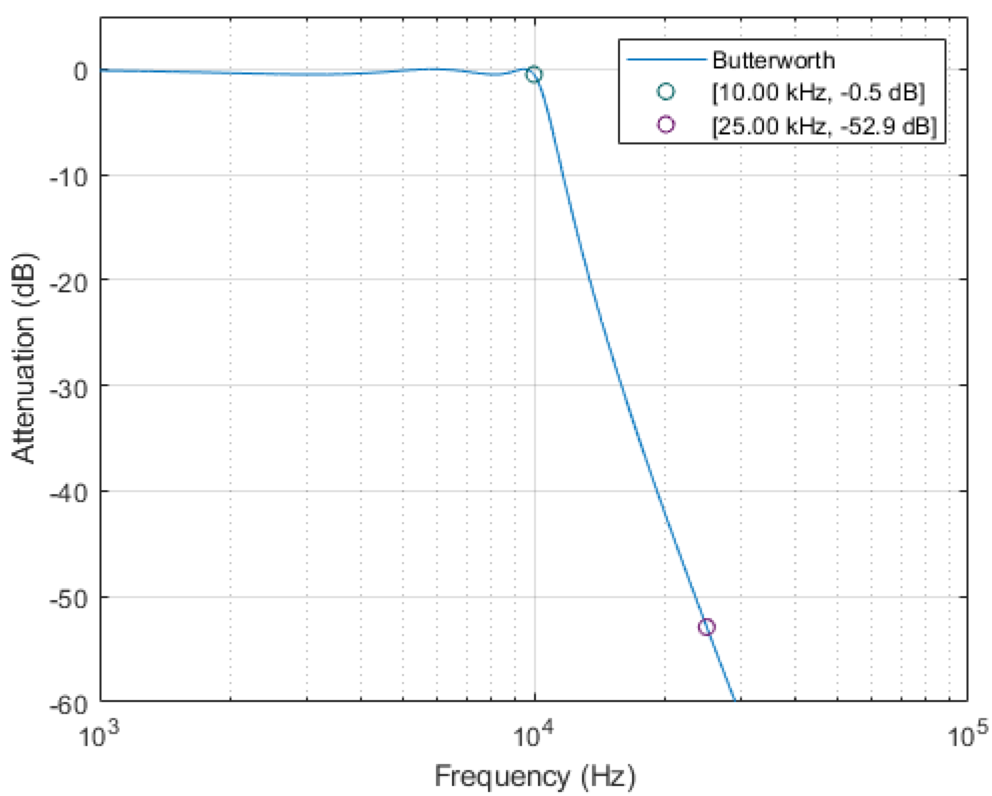
$$= 0.2338 - j1.0332$$

$$\Rightarrow A_v(s=j) = \frac{1}{H(s)} = \frac{1}{0.2338 - j1.0332} = 0.944 \angle 77.249^\circ$$

Phase Shift $\boxed{\theta \cong 77.25^\circ = 0.429\pi \text{ radians} = 1.348 \text{ radians}}$



Problem 1. 5th-order Butterworth Filter Gain Response



Problem 2. 5th-order Chebyshev Filter Gain Response